

Solving Non-deterministic Planning Problems with Pattern Database Heuristics

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- Given:** A non-deterministic planning problem.
(Informally: Initial state, actions, goal states.
Nondeterminism: actions may have several outcomes.)
- Desired:** A solution to that problem.
(Informally: How to reach a goal state, using the actions?)

Given:

Non-deterministic planning problem $\mathcal{P} = (Var, A, s_0, G)$ with:

- Var , finite set of *state variables*.
 $S = 2^{Var}$ is the state space.
- A , finite set of *actions* $a = \langle pre(a), eff(a) \rangle$ and:
 - $pre(a) \subseteq Var$ and
 - $eff(a) = \{ \langle add_i, del_i \rangle \mid add_i, del_i \subseteq Var \text{ and } i \in \{1, \dots, n\} \}$.
 - Its application (if $pre(a) \subseteq s$) leads to:
 $app(s, a) = \{ (s \setminus del) \cup add \mid \langle add, del \rangle \in eff(a) \}$
- $s_0 \in S$, the *initial state*.
- $G \subseteq Var$, the *goal description*.
 A state $s \in S$ is a goal state iff $s \supseteq G$.

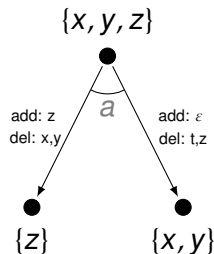
Formalization (Example)

Let $s = \{x, y, z\} \in S$ be a state and $a \in A$ be an action with:

$a = \langle pre(a), eff(a) \rangle$ and

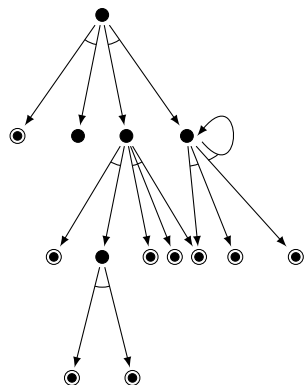
$pre(a) = \{x, y\} \subseteq s$,

$eff(a) = \{ \langle \{z\}, \{x, y\} \rangle, \langle \emptyset, \{t, z\} \rangle \}$.



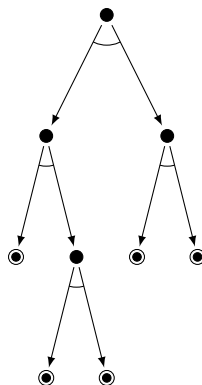
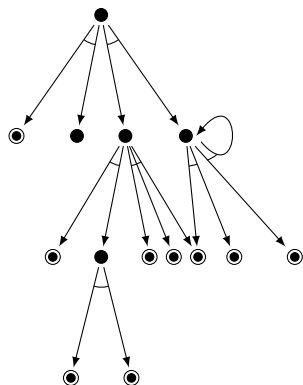
Desired:

Strong plan. (Success, regardless of non-deterministic outcome.)



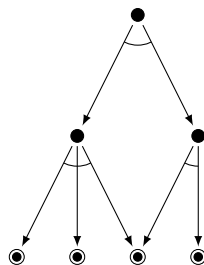
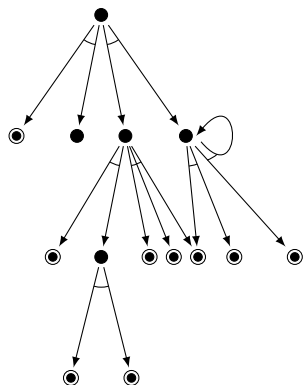
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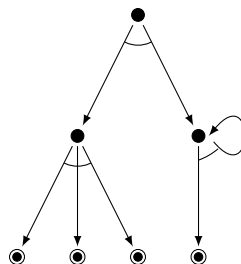
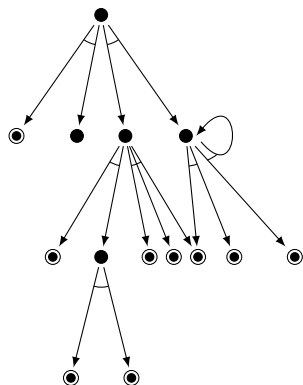
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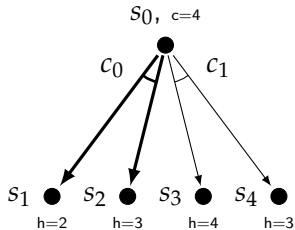


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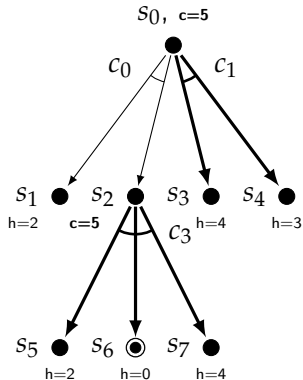
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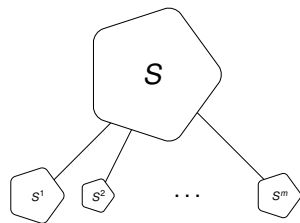
Search Algorithm, modification of AO*



Expansion



Use abstraction to simplify the problem:



Map the search space S to abstract search spaces S^i with $|S^i| \ll |S|$.

Compute $h(s)$, $s \in S$, on basis of all $h^i(s^i)$.

Calculation of the h^i is done *before* the search.

Idea: Disregard some (or rather *most of the*) state variables.

The *abstraction* $\mathcal{P}^i = (Var^i, A^i, s_0^i, G^i)$ is the planning problem \mathcal{P} , restricted to the pattern $P_i \subseteq Var$:

- $Var^i := Var \cap P_i = P_i$,
- For $var \subseteq Var$ let $var^i := var \cap P_i$. Then:
 $a^i := \langle pre(a)^i, \{ \langle add^i, del^i \rangle \mid \langle add, del \rangle \in eff(a) \} \rangle$ for $a \in A$.
 Now, $A^i := \{ a^i \mid a \in A \}$.
- $s_0^i := s_0 \cap P_i$
- $G^i := G \cap P_i$.

Recall:

- A *pattern* is a set of state variables $P_i \subseteq \text{Var}$.
Then, a *pattern collection* P is a set of patterns.
- Compute $h(s), s \in S$, on basis of all $h^i(s^i), P_i \in P, P$ finite pattern collection, i.e. set of patterns.

How to calculate those $h^i(s^i), s^i \in S^i$?

$h^i(s^i)$ is the *true* cost value cost^* of the planning problem \mathcal{P}^i .

Calculation is done by a complete exhaustive search.

(Thus, S^i and therefore P_i have to be small!)

(*True* means: prefer shallow solution graphs.)

Additivity (Theorem)

How to calculate $h(s), s \in S$?

By using additivity!

A pattern collection P is called *additive*, if for all states $s \in S$:

$$\sum_{P_i \in P} h^i(s^i) \leq \text{cost}^*(s), \text{ i.e. if this sum is still admissible.}$$

Known from classical planning:

Theorem (textual description)

If there is no action $a \in A$ that affects variables in more than one pattern from P , then P is additive.

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Known from classical planning:

Theorem (mathematical description)

If for all $a \in A$ and for all patterns $P_i \in P$ holds:

If $P_i \cap \text{effvar}(a) \neq \emptyset$, then $P_j \cap \text{effvar}(a) = \emptyset$ for all $P_j \in P$ with $P_j \neq P_i$,
where $\text{effvar}(a) = \bigcup_{(add,del) \in \text{eff}(a)} \text{add} \cup \text{del}$.

Then P is additive.

Additivity (Example)

$\mathcal{P} = (\{a, b, c, d, e\}, A, \{a\}, \{b, c, d, e\})$ with $A = \{a_1, \dots, a_9\}$ and:

$$a_1 = \langle \{a\}, \{\langle \{b\}, \{a\}\rangle, \langle \{c\}, \{a\}\rangle\} \rangle$$

$$a_6 = \langle \{b, e\}, \{\langle \{c\}, \emptyset\}\rangle \rangle$$

$$a_2 = \langle \{b\}, \{\langle \{e\}, \emptyset\rangle, \langle \{d\}, \emptyset\rangle\} \rangle$$

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Now, consider the pattern collection $P = \{\{a, b, c\}, \{d, e\}\}$.

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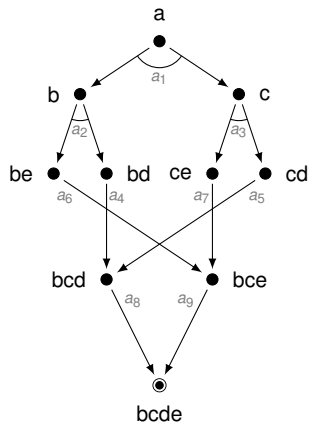
$$a_9 = \langle \{b, c, e\}, \{\langle \{d\}, \emptyset\}\rangle \rangle$$

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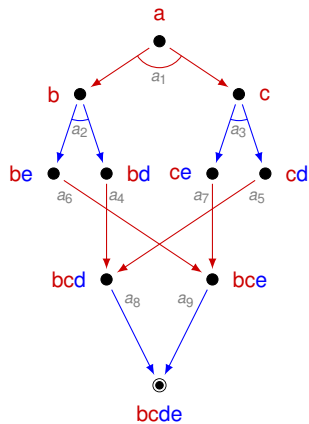
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Only the effect variables matter!

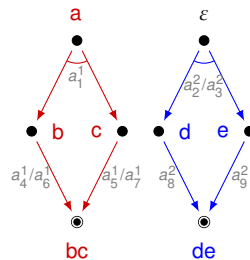
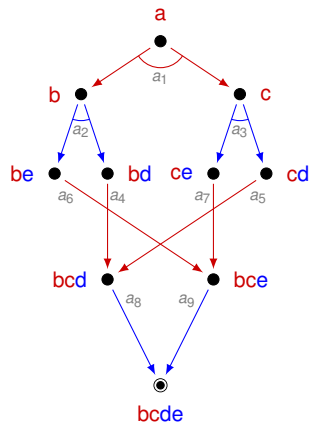
Additivity (Example, cont'd)



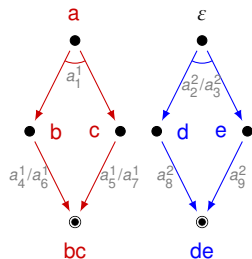
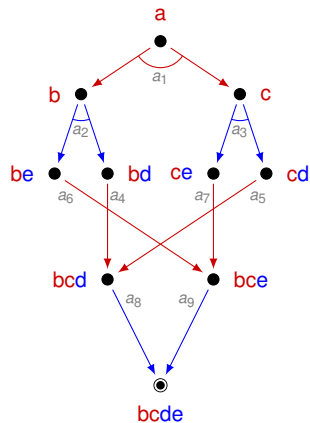
Additivity (Example, cont'd)



Additivity (Example, cont'd)



Additivity (Example, cont'd)



Example:

$$h(\{a\}) = h^1(\{a\}^1) + h^2(\{a\}^2) = h^1(\{a\}) + h^2(\emptyset) = 2 + 2 = 4 = \text{cost}^*(\{a\}).$$

Heuristic Calculation (cont'd)

Let \mathcal{M} be a set of additive pattern collections.

$$h^{\mathcal{M}}(s) := \max_{P \in \mathcal{M}} \sum_{P_i \in P} h^i(s^i).$$

$h^{\mathcal{M}}$ (and in particular, every single h^i) is admissible.

How to find \mathcal{M} ?

Current research. (Here: still domain-dependent by hand.)

Compared Systems

Encoded two domains and compared:

- Our planner with the heuristic of FF.
- Our planner with the presented pattern database heuristics.
- GAMER.

Important differences between GAMER and our system:

- Optimal solutions vs. suboptimal solutions.
- Regression vs. progression.

Results

- Pattern database heuristic quality is problem dependent:
 - Domain 1: Pattern database heuristics about 25% more node expansions than FF heuristic.
 - Domain 2: Pattern database heuristics calculate true (perfect) cost value (as opposed to the FF heuristic).
- Calculation time of pattern database heuristic is much smaller than the FF heuristic's. Thus, more problems could be solved.
- Progression with heuristic search seems promising approach. (Note: No comparison to sub-optimal planner, yet.)

Summary

- Presented formalization for domain-independent pattern database heuristics in non-deterministic planning.
- Generalization of additivity criterion.
- Benchmarks look promising.

- Automatic pattern selection.
- Strong plans → strong *cyclic* plans.
 - Search algorithm, LAO*.
 - Pattern database heuristics: Admissibility/Additivity?
- Multi-valued state variables.

Thank you!